

# The Varying Permeability Model

By Dan Reinders

(with additional graphics and  
animations by Richard Pyle)

An easy explanation for the  
mathematically disinclined

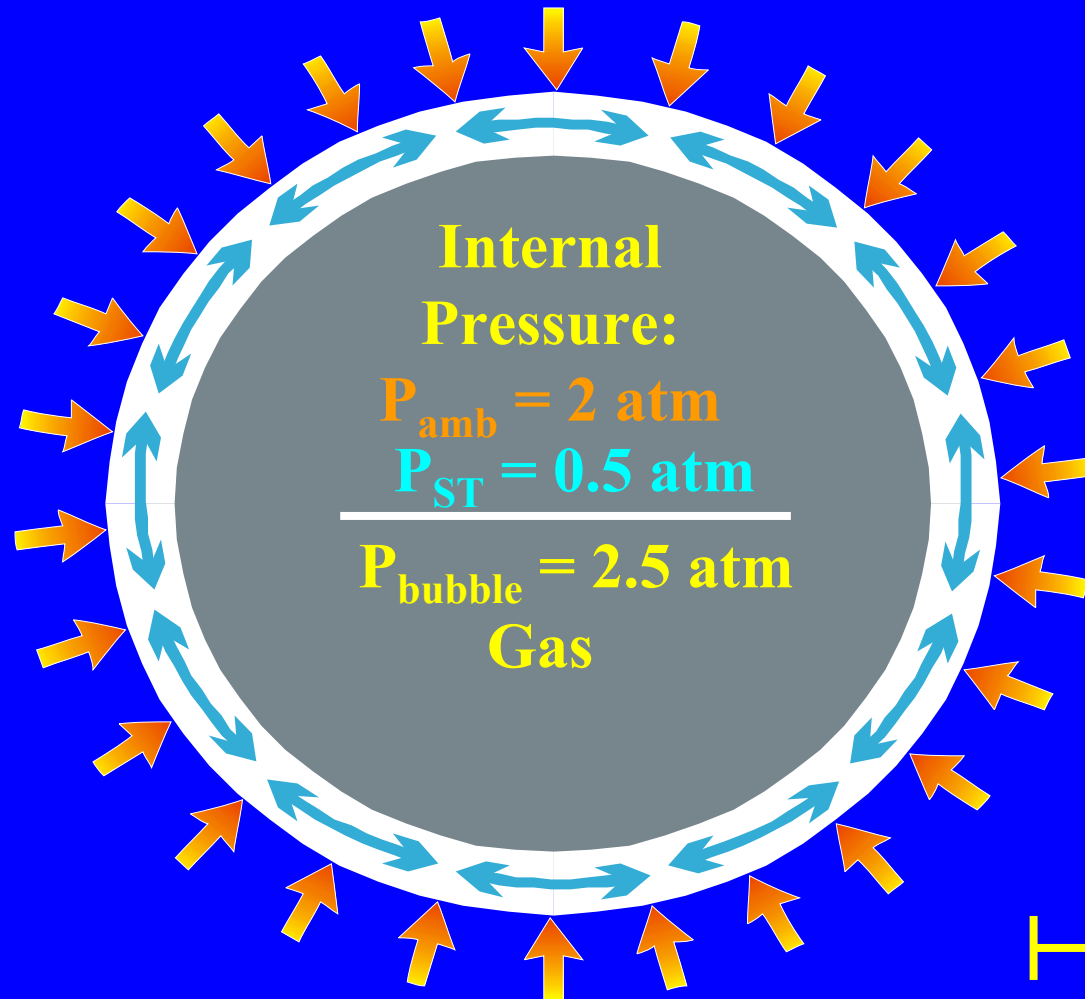
# First an Introduction to bubbles:

- The pressure of gas in a bubble is equal to the surrounding hydrostatic pressure, plus a contribution from the surface tension caused by water molecules “pulling” together at the bubble surface.
- The contribution from surface tension is found by the following formula:  **$P_{ST} = 2\gamma_0/\text{radius}$**
- A bubble about the size of a red blood cell (4  $\mu\text{m}$  radius) has its pressure raised by up to 0.5 atmospheres.
- **The smaller the bubble, the greater the surface tension effect.**

# Effect of Surface Tension

**Ambient Pressure:**  
 $P_{\text{amb}} = 2 \text{ atm}$

**Surface Tension:**  
 $P_{\text{ST}} = 0.5 \text{ atm}$



**Fluid**

$4 \mu\text{m}$

# Gas diffuses from bubbles

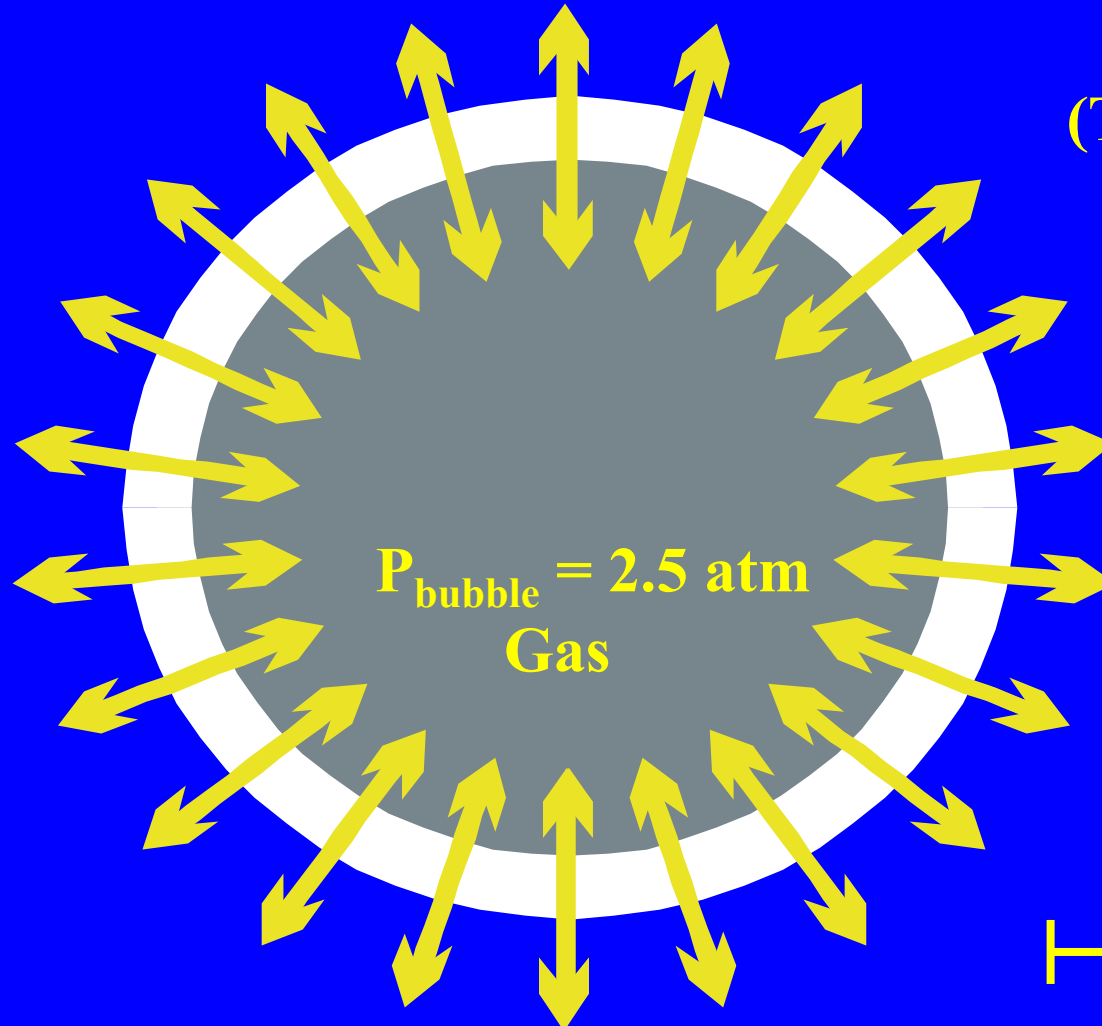
- If the pressure inside a bubble is greater than the pressure of the DISSOLVED gas in the surrounding tissue, the bubble will shrink.
- Conversely if the pressure in the bubble is less than the tissue dissolved gas pressure the bubble will grow.

# Gas Diffusion Gradients

**Ambient Pressure:**  
 $P_{\text{amb}} = 2 \text{ atm}$

**Surface Tension:**  
 $P_{\text{ST}} = 0.5 \text{ atm}$

**Dissolved Gas (Tissue) Pressure:**  
 $P_{\text{tis}} = 3.2 \text{ atm}$



**Fluid**

$4 \mu\text{m}$



# Implications

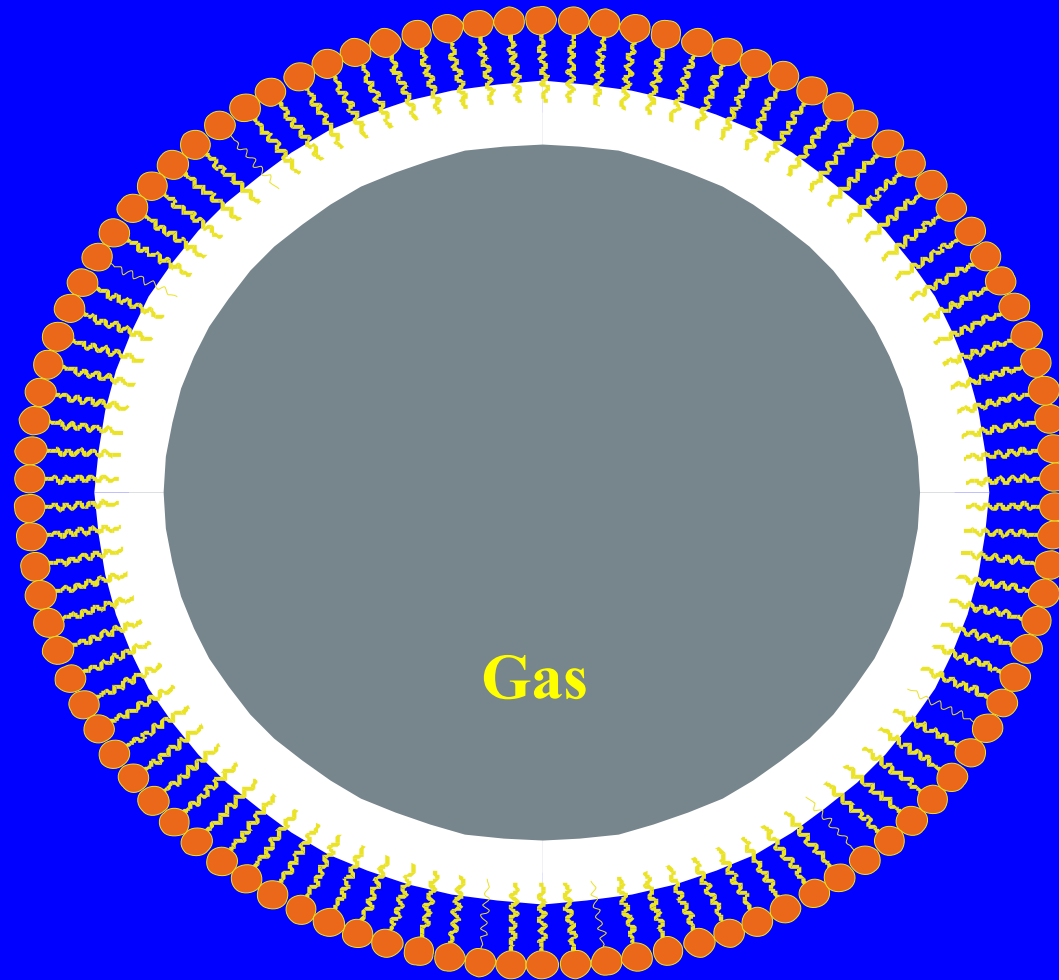
- Except for during decompression, all bubbles should eventually dissolve, because surface tension makes the bubble pressure higher than the surrounding dissolved gas pressure.
- A person who has not been diving recently should not have any bubbles.
- **In reality, bubbles don't always dissolve.**

# Enter the Varying Permeability Model!

- To explain why bubbles don't always dissolve, a lot of ideas have been suggested.
- The best explanation so far is that the tiny bubbles become stabilized by “surface active molecules”
- These are molecules that have both a *hydrophobic* component and a *hydrophilic* component, and embed themselves in the gas-water interface.

# Surface Active Molecules

**Hydrophilic** [  ]  
**Hydrophobic** [  ]

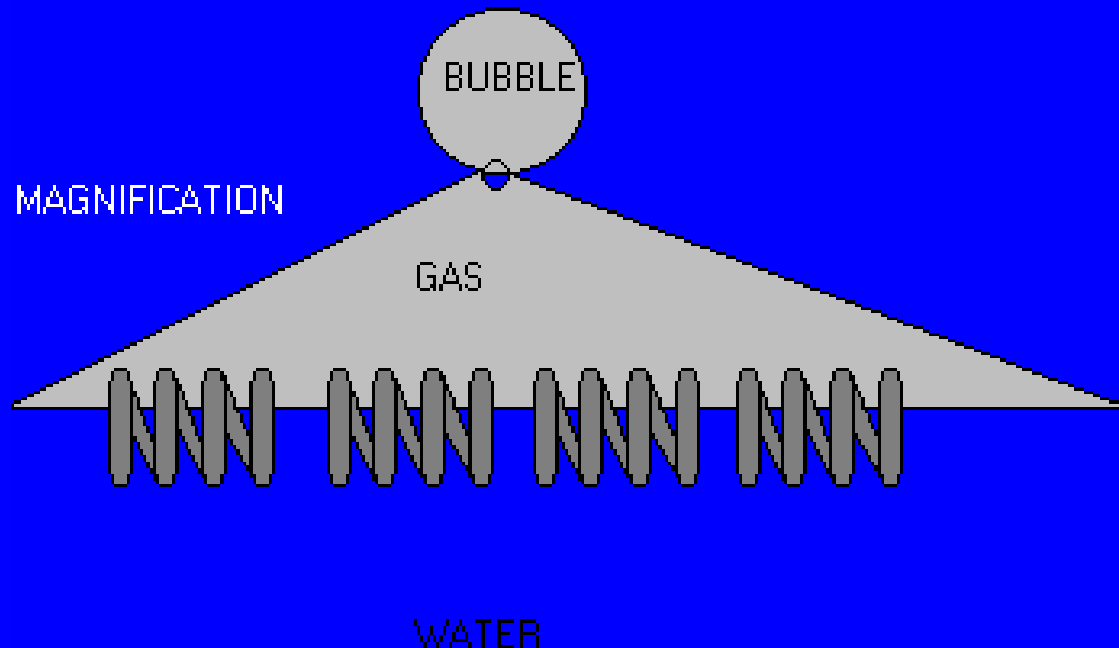


**Fluid**

# How do “surfactants” stabilize bubbles?

- Just as the water molecules “pull” towards each other in surface tension, the surface active molecules “push” against each other.
- This counteracts the effect of surface tension, and therefore eliminates the loss of gas by diffusion.
- No diffusion means no bubble dissolution.

Surfactants can be thought of as tiny springs pushing against each-other at the interface.

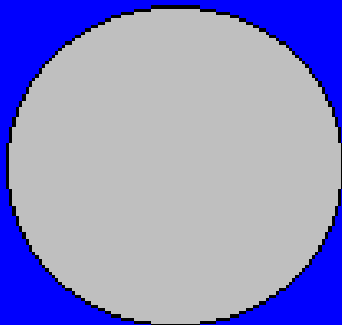


# What happens during crushing?

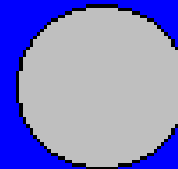
- When a bubble is compressed by descending, the area available for each spring lowers. Basically each spring compresses as it bumps against it's neighbors.
- But just like a real spring, eventually it can't compress any more - it runs out of travel.
- At this point springs will start popping off the bubble surface.

- More precisely, it becomes energetically favorable for a surfactant molecule to leave the surface rather than to compress further.
- The effect of surface tension is now countered and the bubble stabilizes at its new smaller radius.

PRE-COMPRESSION BUBBLE



POST-COMPRESSION BUBBLE



# Growing Bubbles

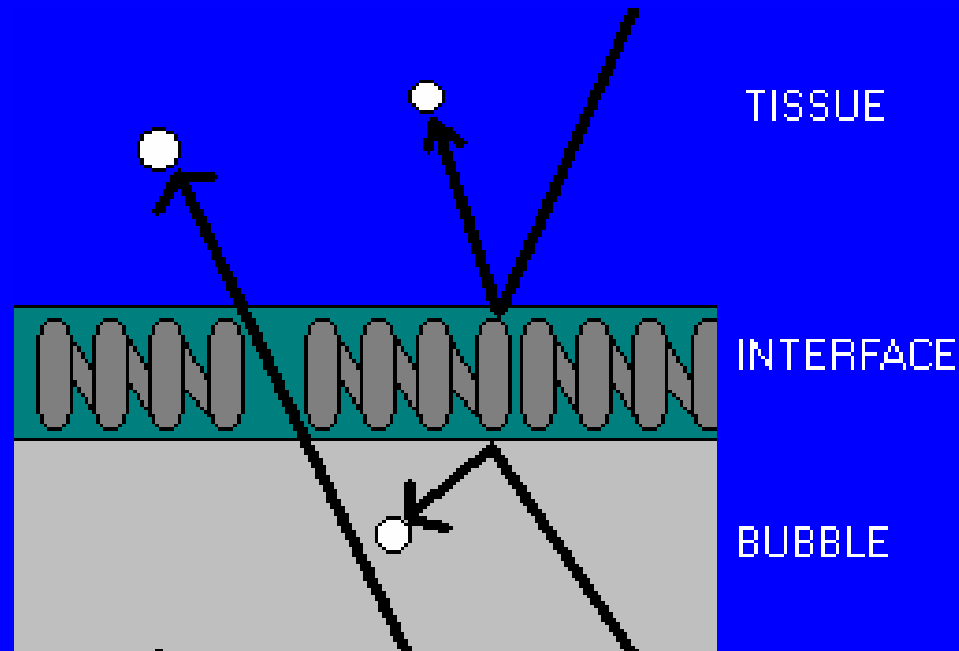
- Recall that bubbles grow when the dissolved gas pressure is greater than the interior bubble pressure.
- This means that small bubbles require a greater “super-saturation” in order to be stimulated into growth, because the effect of surface tension is proportionally greater for smaller bubbles.
- Therefore crushed nuclei are better for divers than uncrushed nuclei.

Wait a second - didn't you just say that the surface tension was negated in the crushed nuclei?

- This would mean that small bubbles should grow just as easily as large bubbles.
  - **But this doesn't happen!** –
- At first the bubble expands, but then the springs “lose contact” with each-other, so they can't push against each-other, and the effect from the surfactant molecules is lost.
- Thus, surface tension reigns supreme.

# Do surfactants have any other effects?

- Yes - they form a barrier to diffusion.
- The closer they are squeezed together, the stronger the barrier to diffusion.



# Kunkle vs. Yount

- There are two main bubble surfactant models out there:
- One by Dr. Thomas Kunkle
- One by Dr. David Yount

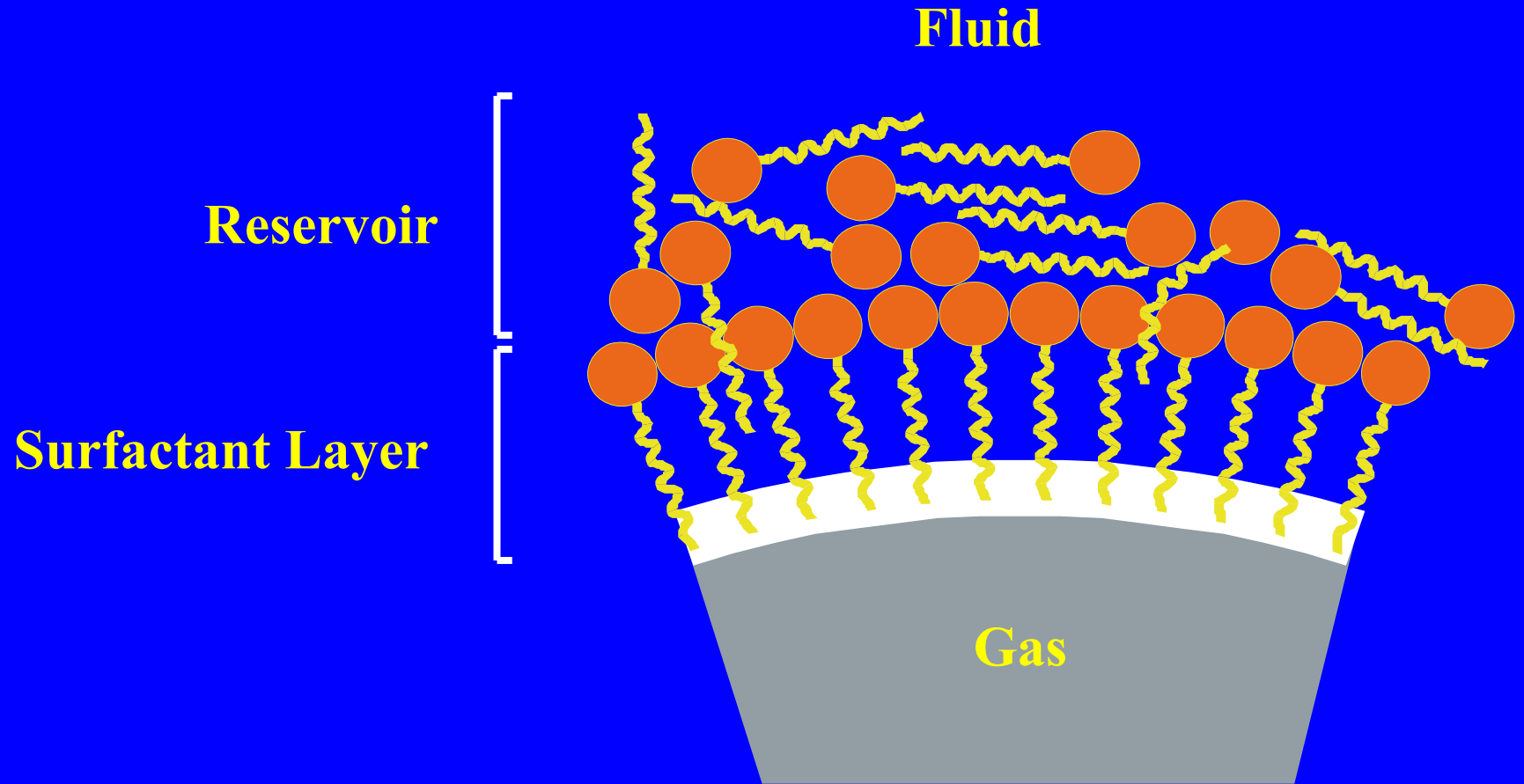
# Kunkle's model

- Assumes that when surfactants leave the bubble they don't return or interact in any way.
- Fully accounts for the “springiness” of the springs.
- The diffusion barrier strength depends on the space available for each surfactant.

# Yount's Model

- Assumes that there is a reservoir of surfactants “hanging around” just outside the bubble.
- Accounts for the transfer of surfactant molecules between the reservoir and the bubble surface.
- Uses “unspringy springs”, the springs either don't push back or else push back at their “popping-off” threshold. They act more like billiard balls than springs.

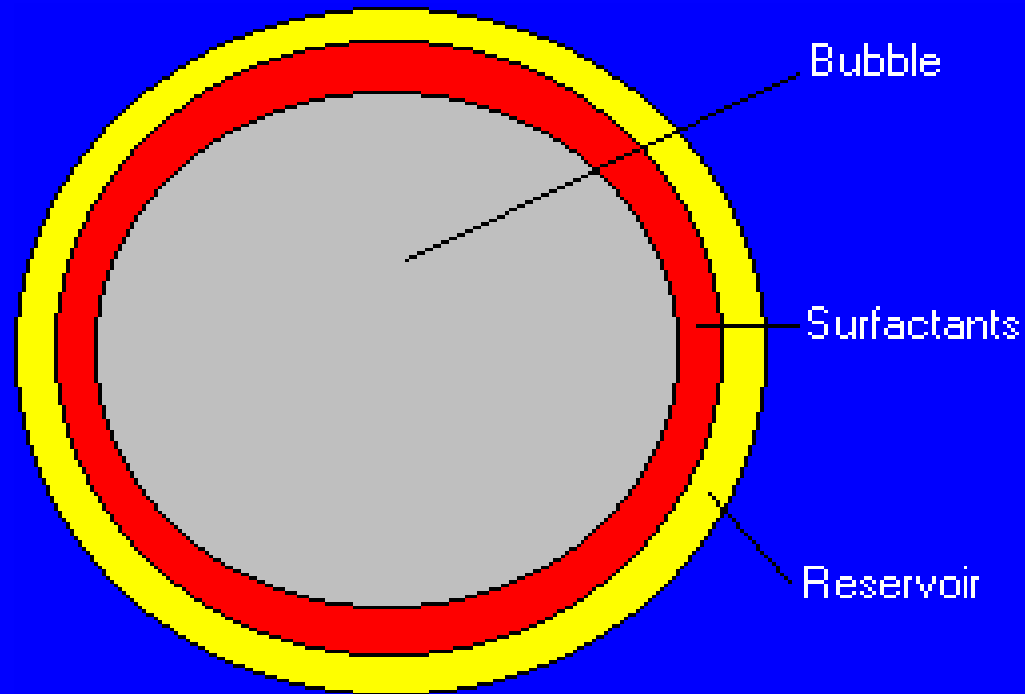
# Surfactant Reservoir



# What's the deal with “Varying Permeability”?

- The surfactants either don't form a diffusion barrier, or completely block diffusion.
- This “impermeability” occurs after about 300 fsw of compression, so is not really a concern for most divers.
- An impermeable bubble won't be crushed as much as a permeable bubble because gas doesn't diffuse out as it shrinks.

# The Reservoir



- The VPM also accounts for an electrostatic force between the reservoir and the surface.

# The Electrostatic Forces

- The pressure balance equation is:

$$\mathbf{P}_{\text{bubble}} + (2 \gamma_{\text{c}}/\text{radius}) - \mathbf{B} \\ = \mathbf{P}_{\text{amb}} + 2 \gamma_{\text{o}}/\text{radius}$$

- **B** is the sum of various electrical and chemical attractions and repulsions.
- $\gamma_{\text{c}}$  accounts for the springy “push back” effect of the surfactants.

# What we need to know about bubble crushing.

- We assume that the gas pressure in the bubble is equal to the outside tissue pressure - aka diffusive equilibrium.
- Ignoring oxygen effects, this means that  $P_{\text{bubble}}$  is equal to  $P_{\text{ambient}}$ , because  $P_{\text{ambient}}$  would equal the dissolved gas pressure ( $P_{\text{dis}}$ ).

- Using the pressure equation:

before crushing:

$$2 \gamma_c / r_0 - B_0 = P_{\text{surface}} + 2 \gamma_c / r_0$$

$$P_{\text{tis}} +$$

after crushing:

$$P_{\text{tis}} + 2 \gamma_c / r_{\text{crush}} - B_{\text{crush}} = P_{\text{depth}} + 2 \gamma_c / r_{\text{crush}}$$

- Where  $P_{\text{tis}}$  is the dissolved gas pressure (assumed equal to  $P_{\text{surface}}$ ),  $r_0$  is the initial radius, and  $r_{\text{crush}}$  is the final radius.
- Setting  $B_0$  equal to  $B_{\text{crush}}$  gives us the equation for the crushed radius.

# The CRUSHING formula:

$$P_{\text{crush}} = P_{\text{depth}} - P_{\text{tis}}$$

$$\text{CF} = \text{Crush factor} = 2 (\gamma_{\text{oc}} - \gamma_{\text{o}})$$

$$r_{\text{crush}} = 1 / \left( \left( \frac{P_{\text{crush}}}{\text{CF}} \right) + \frac{1}{r_{\text{o}}} \right)$$

# The Meta-Stable state

- A different **B** value is used as the tissue saturates, to represent the nuclei forming a semi-stable state.
- The nuclei are exponentially restored to their original size as surfactants return from the reservoir to the interface.
- This process occurs over many days, but may occur faster in living organisms.

# Decompression and Nuclei

- Even a bubble not stimulated to growth will expand with a drop in ambient pressure.
- The same equations are used:

During saturation:

$$P_{\text{dis}} + 2 \gamma_{\text{c}}/r_{\text{s}} - B_{\text{s}} = P_{\text{depth}} + 2 \gamma_{\text{o}}/r_{\text{s}}$$

After decompression:

$$P_{\text{dis}} + 2 \gamma_{\text{c}}/r_{\text{d}} - B_{\text{d}} = P_{\text{surface}} + 2 \gamma_{\text{o}}/r_{\text{d}}$$

- The **s** subscript refers to saturation, **d** refers to decompression.

# Bubble Growth

- Bubbles grow when the super-saturation pressure is greater than  $2 \gamma_b / \text{radius}$  (surface tension).
- Note that nuclei growth during decompression makes it easier for nuclei to evolve into full-fledged bubbles.
- All of the previous equations can be combined to find the smallest bubble stimulated into growth.

# Bubble Numbers:

- The VPM predicts that there is an exponential distribution of nuclei - lots of small ones and a few big ones.
- The number of nuclei stimulated into growth is related to the minimum size stimulated into growth by the following equation:

$$N_{\text{stimulated}} = N_{\text{total}} (e^{-K * r_{\text{stimulated}}})$$

# Take Home Messages

- Greater super-saturation stimulates more bubbles into growth
- Greater crushing pressures help minimise the number of stimulated bubbles
- Saturation decompressions must be more conservative to allow for the loss of the crushing effects.

# VPM and dive tables

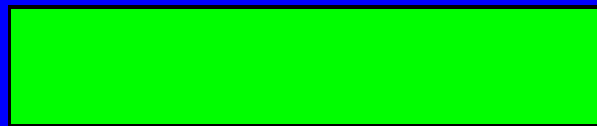
- There is a lot of confusion about how the VPM is integrated in dive models.
- The concept is actually quite simple, but this simplicity is somewhat hidden by the elegant procedures used to generate the dive tables.

# Minimum Bubble Number

- The VPM assumes that there is a minimum bubble number (regardless of bubble size) that can be tolerated without decompression sickness.
- If this is true, then keeping the super-saturation below that required to stimulate the critical number of nuclei should prevent decompression sickness.

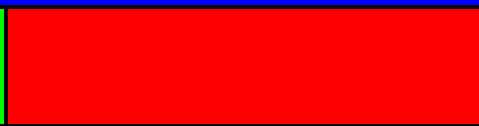
- This assumption works great for saturation exposures, but is too conservative for normal (no-deco/mild deco) dives.
- Solution - assume that there is a maximum volume of gas that is allowed, **ONLY** counting nuclei from below a critical radius

Nuclei larger than the critical radius



Ignore the volume of gas bubbles from nuclei above the critical radius.

Nuclei smaller than the critical radius



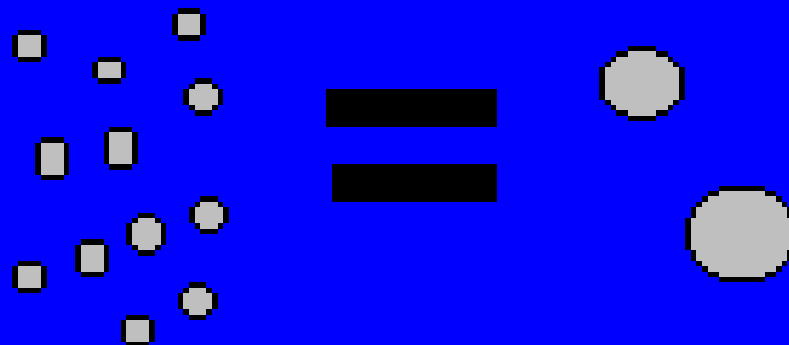
Make sure the volume of gas FROM THESE BUBBLES ONLY is less than the maximum permitted volume

# Half-times and bubble growth

- “Fast tissues” remove inert gas faster than slow tissues, meaning that bubbles don’t have time to grow as big as they do in slow tissues.
- Initially the bubbles grow faster because of the typically higher pressure difference, but this is greatly outweighed by the quick removal of source gas.

# Many small or few big

- This critical volume concept means fast tissues can have lots of small bubbles, while slow tissues can have hardly any bubbles above the minimum number.
- A greater super-saturation is allowed for fast tissues.



# Increasing Gradients

- The VPM starts out by just stimulating the minimum safe number of bubbles.
- The maximum allowed super-saturation is then increased, and the volume of excess gas in each compartment is compared to the maximum permitted.
- If it is less than allowed, the super-saturation is increased again and again, until the compartment maximum is reached.

# Does the VPM apply?

- It certainly has shown that it can be used to generate successful dive tables.
- It has some support from human and animal data.
- It has apparently been successful during data fitting by Dr. Wienke with the new Reduced Gradient Bubble Model.

# Other candidate models

- Many of the successes of the VPM (deeper predicted decompression stops, etc) can also be explained by models of diffusive bubble growth and “phase equilibrium” models (where there is an excess of available nuclei for the gas to grow into bubbles).
- Impossible at present to tell which model is correct, so best to reserve judgement.

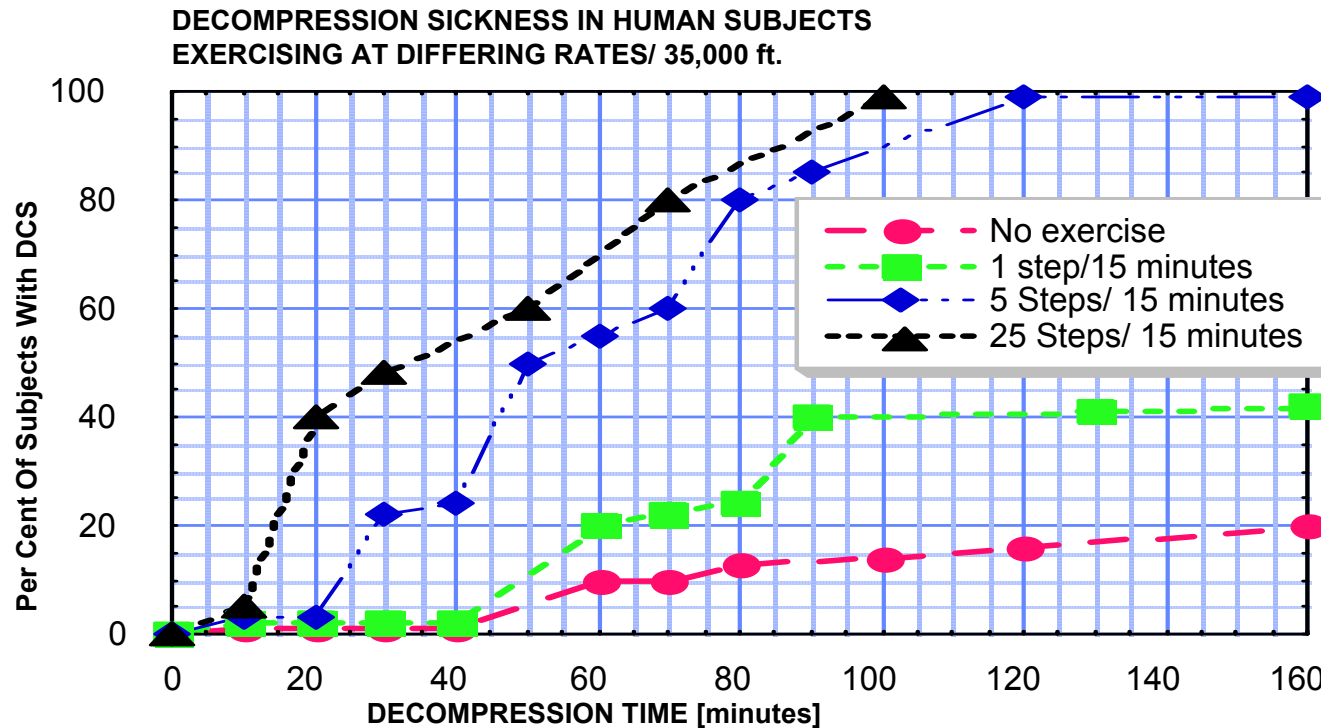
# Other ways to stabilize nuclei

- Hydrophobic crevices can also form nuclei (you see this in your beer glass).
- It is clear that nuclei are NOT forming out of pure water, that would require supersaturation's greater than the depth of the ocean.
- Short-lived nuclei can also be continuously created by stresses in muscles and joints. These nuclei may only be partially stabilized, and have a life-span of minutes to hours

# Creation of new nuclei

- Note that the VPM doesn't deal with creation of new nuclei, only with the stabilization of those created nuclei.
- These nuclei, created by stresses during movement, are likely to be the reason why decompression sickness is more likely if exercise is performed prior to or during decompression.

# Exercise and nuclei



After Ferris et al., Committee on Aviation Medicine, Report 363, 1943

The effect of exercise is shown above, aviators who did step exercises had greater incidences of decompression sickness.

# Does that mean I should rest on the deco-line?

- Not necessarily.
- It is true that exercise creates more nuclei.
- But exercise also increases the removal of gas from tissues, and has been found to be beneficial in some studies.
- What is clear is that strenuous exercise must be avoided.
- It is not clear when and where mild exercise should or should not be performed

# Model of the Future?

- Current evidence suggests that both stabilized, VPM style nuclei and short lived, movement generated nuclei are important in decompression sickness.
- These two effects operate on long term and short term time scales.
- The model of the future will likely account for both of these effects

# Bottom line

- Both phase equilibrium, diffusive bubble growth and VPM models have been used to successfully generate dive tables.
- All of these models make suggestions of the same nature (deeper stops and lower supersaturations), so we don't really have a way to discriminate amongst them.
- But because they make similar suggestions, any of these bubble models is likely to be superior to standard "Haldanean" tables.

# Conclusion

- VPM recommendations make sense from a variety of perspectives.
- Surfactant stabilized micronuclei may or may not prove to be a key player in human decompression sickness, but regardless the pioneering work of Kunkle and Yount has greatly broadened our understanding of how bubbles form and stabilize - their contribution should not be underestimated.